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Heteromorphe Abbildungen bei dirempten Trajekten

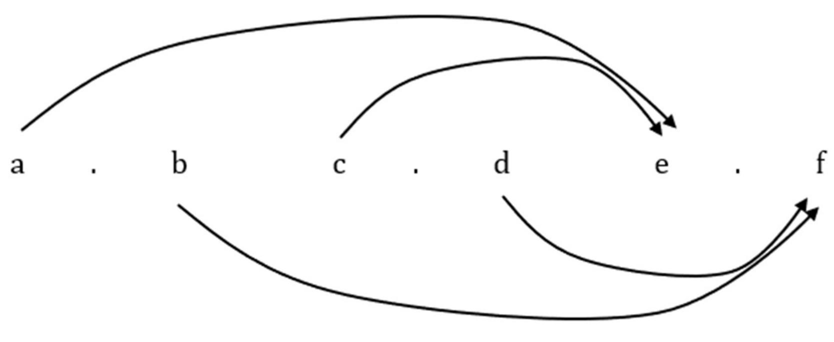
1. Weil die Vorstellung der in der klassischen Kategorientheorie unbekannten heteromorphen Abbildungen immer wieder zu Mißverständnissen führt, sei hier Rudolf Kaehr, der sie entdeckt hat, etwas ausführlicher zitiert:

> Morphisms are representing mappings between objects, seen as domains and codomains of the mapping function.
>
> Hetero-morphisms are representing the conditions of the possibility (Bedingungen der Möglichkeit) of the composition of morphisms. That is, the conditions, expressed by the matching conditions, are reflected at the place of the hetero-morphisms. Hetero-morphisms as reflections of the matching conditions of composition are therefore *second-order* concepts. Morphisms and their composition are *first-order* concepts, which have to match the matching conditions defined by the axiomatics of the categorical composition of morphisms. But these matching conditions are nor explicit in the composition of morphism but implicit, defined "outside" of the compositional system. Hence, in diamonds, the matching conditions of categories are explicit, and moved from the "outside" into the inside of the system.
>
> In this sense, the rejectional system of hetero-morphisms is a reflectional system, reflecting the interactions of the compositions of the acceptional system. Hetero-morphisms are, thus, the "morphisms" of the matching conditions for morphisms.
>
> Hetero-morphisms are "composed" by the jump operation, which is not interactional in the sense of the acceptional system.

(Kaehr 2007, S. 17)

2. Heteromorphe Abbildungen bei dirempten Trajekten

2.1. Rechtsgerichtete dirempte Trajektionen



$T^{dir\rightarrow}(abcdef) = (aecebfdf)$

$T^{dir\rightarrow}(3.1, 2.1, 1.1) = (3.1, 2.1, 1.1, 1.1)$ $(1 \leftarrow 2), (1 \leftarrow 1), (1 \leftarrow 1)$

$T^{dir\rightarrow}(3.1, 2.1, 1.2) = (3.1, 2.1, 1.2, 1.2)$ $(1 \leftarrow 2), (1 \leftarrow 1), (2 \leftarrow 1)$

$T^{dir\rightarrow}(3.1, 2.1, 1.3) = (3.1, 2.1, 1.3, 1.3)$ $(1 \leftarrow 2), (1 \leftarrow 1), (3 \leftarrow 1)$

$T^{dir\rightarrow}(3.1, 2.2, 1.2) = (3.1, 2.1, 1.2, 2.2)$ $(1 \leftarrow 2), (1 \leftarrow 1), (2 \leftarrow 2)$

$T^{dir\rightarrow}(3.1, 2.2, 1.3) = (3.1, 2.1, 1.3, 2.3)$ (1 ← 2), (1 ← 1), (3 ← 2)

$T^{dir\rightarrow}(3.1, 2.3, 1.3) = (3.1, 2.1, 1.3, 3.3)$ (1 ← 2), (1 ← 1), (3 ← 3)

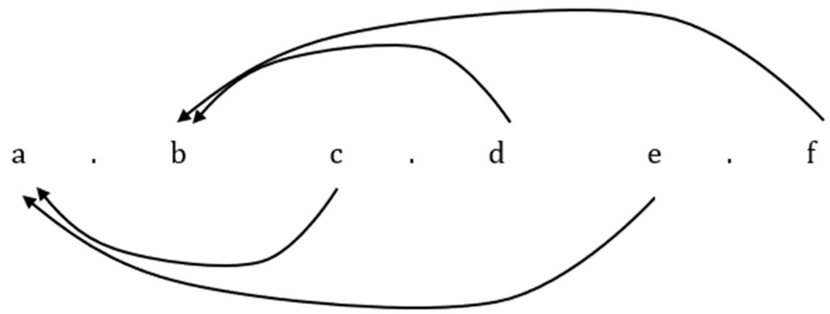
$T^{dir\rightarrow}(3.2, 2.2, 1.2) = (3.1, 2.1, 2.2, 2.2)$ (1 ← 2), (1 ← 2), (2 ← 2)

$T^{dir\rightarrow}(3.2, 2.2, 1.3) = (3.1, 2.1, 2.3, 2.3)$ (1 ← 2), (1 ← 2), (3 ← 2)

$T^{dir\rightarrow}(3.2, 2.3, 1.3) = (3.1, 2.1, 2.3, 3.3)$ (1 ← 2), (1 ← 2), (3 ← 3)

$T^{dir\rightarrow}(3.3, 2.3, 1.3) = (3.1, 2.1, 3.3, 3.3)$ (1 ← 2), (1 ← 3), (3 ← 3)

2.2. Linksgerichtete dirempte Trajektionen



$T^{dir\leftarrow}(abcdef) = (fbdbeaca)$

$T^{dir\leftarrow}(3.1, 2.1, 1.1) = (1.1, 1.1, 1.3, 2.3)$ (1 ← 1), (1 ← 1), (3 ← 2)

$T^{dir\leftarrow}(3.1, 2.1, 1.2) = (2.1, 1.1, 1.3, 2.3)$ (1 ← 1), (1 ← 1), (3 ← 2)

$T^{dir\leftarrow}(3.1, 2.1, 1.3) = (3.1, 1.1, 1.3, 2.3)$ (1 ← 1), (1 ← 1), (3 ← 2)

$T^{dir\leftarrow}(3.1, 2.2, 1.2) = (2.1, 2.1, 1.3, 2.3)$ (1 ← 2), (1 ← 1), (3 ← 2)

$T^{dir\leftarrow}(3.1, 2.2, 1.3) = (3.1, 2.1, 1.3, 2.3)$ (1 ← 2), (1 ← 1), (3 ← 2)

$T^{dir\leftarrow}(3.1, 2.3, 1.3) = (3.1, 3.1, 1.3, 2.3)$ (1 ← 3), (1 ← 1), (3 ← 2)

$T^{dir\leftarrow}(3.2, 2.2, 1.2) = (2.2, 1.1, 1.3, 2.3)$ (2 ← 1), (1 ← 1), (3 ← 2)

$T^{dir\leftarrow}(3.2, 2.2, 1.3) = (3.2, 2.2, 1.3, 2.3)$ (2 ← 2), (2 ← 1), (3 ← 2)

$T^{dir\leftarrow}(3.2, 2.3, 1.3) = (3.2, 3.2, 1.3, 2.3)$ (2 ← 3), (2 ← 1), (3 ← 2)

$T^{dir\leftarrow}(3.3, 2.3, 1.3) = (3.3, 3.3, 1.3, 2.3)$ (3 ← 3), (3 ← 1), (3 ← 2)

Während also die Abbildung von Heteromorphismen auf rechtsgerichtete dirempte Trajektionen bijektiv ist, ist diejenige auf linksgerichtete nicht-bijektiv. So haben etwa die ersten drei Trajektionen identische heteromorphe Abbildungen:

$T^{dir\leftarrow}(3.1, 2.1, 1.1) = (1.1, 1.1, 1.3, 2.3)$ (1 ← 1), (1 ← 1), (3 ← 2)

$T^{dir\leftarrow}(3.1, 2.1, 1.2) = (2.1, 1.1, 1.3, 2.3)$ $(1 \leftarrow 1), (1 \leftarrow 1), (3 \leftarrow 2)$

$T^{dir\leftarrow}(3.1, 2.1, 1.3) = (3.1, 1.1, 1.3, 2.3)$ $(1 \leftarrow 1), (1 \leftarrow 1), (3 \leftarrow 2)$

Literatur

Kaehr, Rudolf, The Book of Diamonds. Glasgow, U.K. 2007

Toth, Alfred, Dirempte Trajektionen. In: Electronic Journal for Mathematical Semiotics, 2026

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